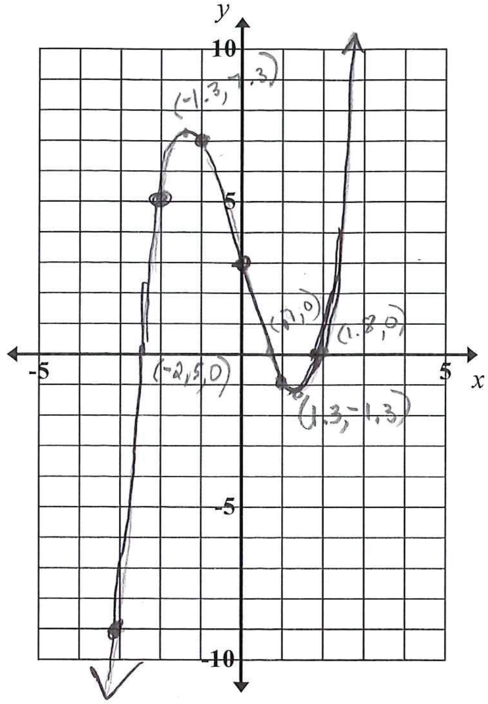
Name ____________________ Period __________

6.1B Graphing Polynomial Functions: Significant Features

#1 – 2: Use a table of values to graph each equation and identify the significant features of the graph.

1. $y = x^3 - 5x + 3$

x	y
-3	-9
-2	5
-1	7
0	3
1	-1
2	1
3	15



Sign of the Lead Coefficient: Positive

End behavior: ↙↗

Domain: all reals

Range: all reals

Relative minimum: (1.3, -1.3)

Relative maximum: (-1.3, 7.3)

Interval(s) where function values are increasing: $x < -1.3$, $x > 1.3$

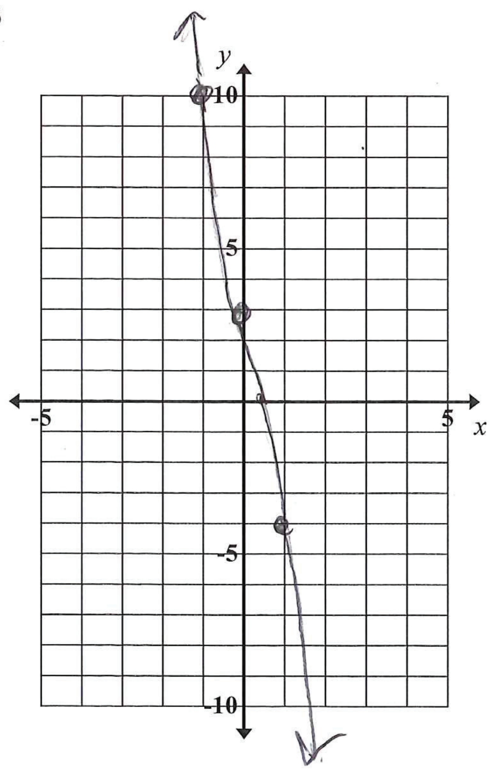
Interval(s) where function values are decreasing: $-1.3 < x < 1.3$

x-intercepts(s): (-2.5, 0), (0.7, 0), (1.8, 0)

y-intercept(s): (0, 3)

2. $y = -2x^3 - 5x + 3$

x	y
-2	29
-1	10
0	3
1	-4
2	-23



Sign of the Lead Coefficient: Negative

End behavior: ↖↘

Domain: all reals

Range: all reals

Relative minimum: none

Relative maximum: none

Interval(s) where function values are increasing: none

Interval(s) where function values are decreasing: $-\infty < x < \infty$

x-intercepts(s): (0.54, 0)

y-intercept(s): (0, 3)

3. How does a cubic function in standard form $y = ax^3 + bx^2 + cx + d$, relate to the significant features of the graph? a determines the end behavior; if $a > 0$, end beh is ↙↗; if $a < 0$, end beh ↖↘. d is the y-intercept

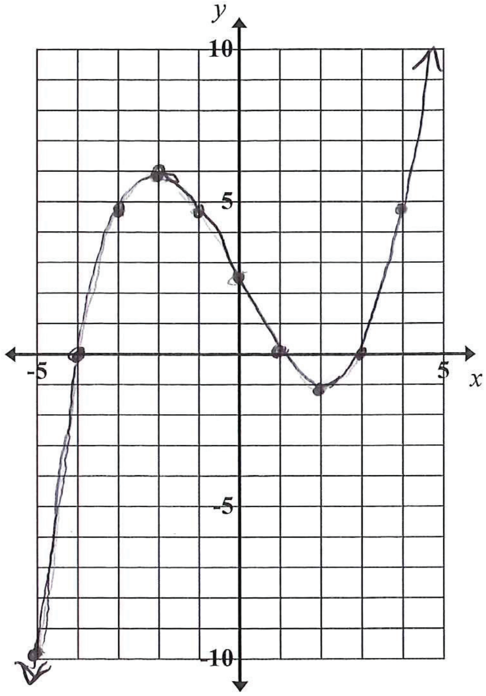
Name ______________________ Period __________

6.1B Graphing Polynomial Functions: Significant Features

#4 – 5: Use a table of values to graph each equation and identify the significant features of the graph.

4. $y = 0.2(x+4)(x-3)(x-1)$

x	y
-5	-9.6
-4	0
-3	4.8
-2	6
-1	4.8
0	2.4
1	0
2	-1.2
3	0
4	4.8



Sign of the Lead Coefficient: positive

End behavior: ↙↗

Domain: all reals

Range: all reals

Relative minimum: (2, -1.2)

Relative maximum: (-2, 6)

Interval(s) where function values are increasing: $x < -2$, $x > 2$

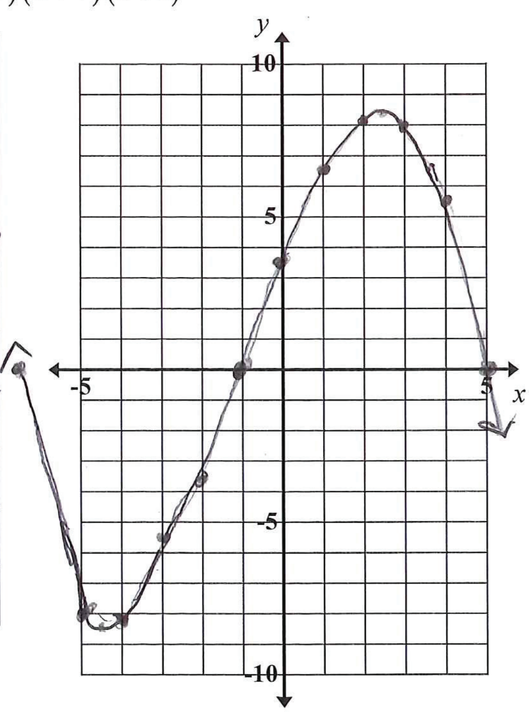
Interval(s) where function values are decreasing: $-2 < x < 2$

x-intercepts(s): (-4, 0), (1, 0), (3, 0)

y-intercept(s): (0, 2.4)

5. $y = -0.1(x-5)(x+7)(x+1)$

x	y
-5	-8
-4	-8.1
-3	-6.4
-2	-3.5
-1	0
0	3.5
1	6.4
2	8.1
3	8
4	5.5
5	0



Sign of the Lead Coefficient: negative

End behavior: ↖↘

Domain: all reals

Range: all reals

Relative minimum: (-4.5, -8.3)

Relative maximum: (2.5, 8.3)

Interval(s) where function values are increasing: $-4.5 < x$ 2.5

Interval(s) where function values are decreasing: $x < -4.5$, $x > 2.5$

x-intercepts(s): (-7, 0), (-1, 0), (5, 0)

y-intercept(s): (0, 3.5)

6. How does a cubic function in factored form $y = a(x-m)(x-n)(x-p)$, relate to the significant features of the graph? a determines the end behavior; if $a > 0$, ↙↗; if $a < 0$, ↖↘
m, n, p are the x-intercepts
$a(-m)(-n)(-p)$ = y intercept

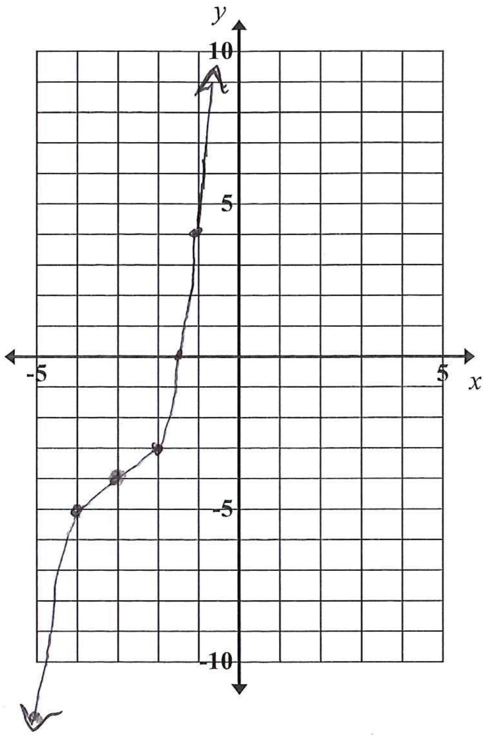
Name ______________________ Period __________

6.1B *Graphing Polynomial Functions: Significant Features*

#7 – 8: Use a table of values to graph each equation and identify the significant features of the graph.

7. $y=(x+3)^3-4$

x	y
-5	-12
-4	-5
-3	-4
-2	-3
-1	4
0	23
1	60
2	121



Sign of the Lead Coefficient: positive

End behavior: ↙↗

Domain: all reals

Range: all reals

Relative minimum: none

Relative maximum: none

Interval(s) where function values are increasing: $-\infty < x < \infty$

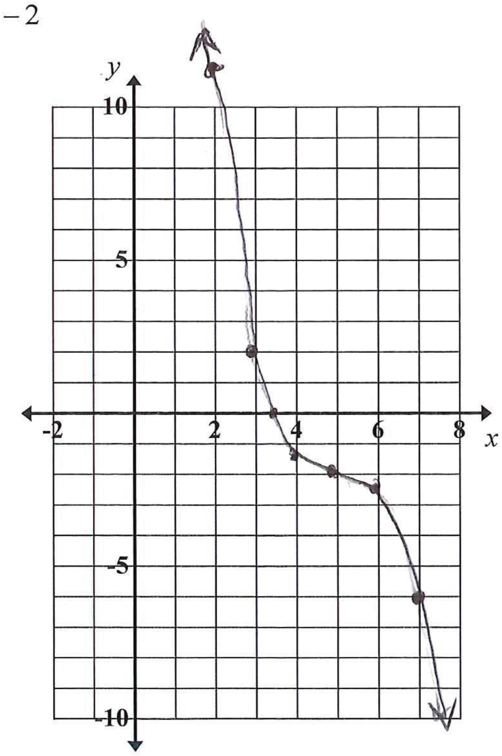
Interval(s) where function values are decreasing: none

x-intercepts(s): $(-1.4, 0)$

y-intercept(s): $(0, 23)$

8. $y=-\frac{1}{2}(x-5)^3-2$

x	y
0	60.5
1	30
2	11.5
3	2
4	-1.5
5	-2
6	-2.5
7	-6
8	-15.5



Sign of the Lead Coefficient: negative

End behavior: ↖↘

Domain: all reals

Range: all reals

Relative minimum: none

Relative maximum: none

Interval(s) where function values are increasing: none

Interval(s) where function values are decreasing: $-\infty < x < \infty$

x-intercepts(s): $(3.4, 0)$

y-intercept(s): $(0, 60.5)$

9. How does a cubic function in the form $y=a(x-h)^3+k$, relate to the significant features of the graph?

"a" determines the end behavior; there is only one x-intercept; also with no rel min or rel max, the function values are either always increasing or always decreasing. So there are no turning points.

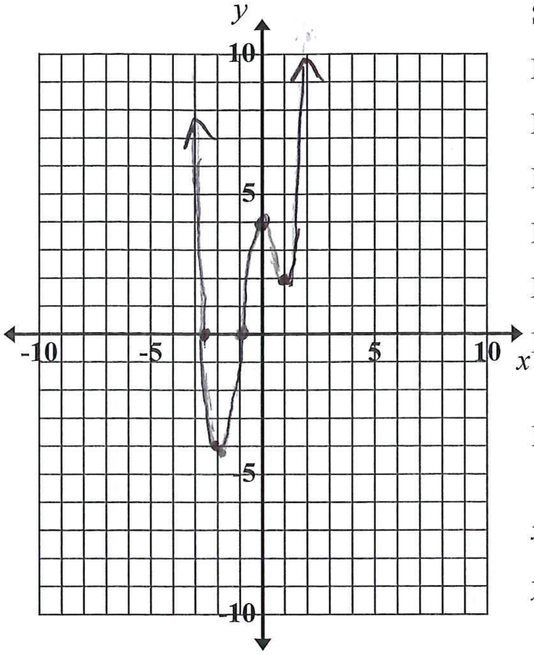
Name ______________________ Period __________

6.1B *Graphing Polynomial Functions: Significant Features*

#10 – 11: Use a table of values to graph each equation and identify the significant features of the graph.

10. $y = x^4 + x^3 - 4x^2 + 4$

x	y
-3	22
-2	-4
-1	0
0	4
1	2
2	12
3	76



Sign of the Lead Coefficient: positive

End behavior: ↖ ↗

Domain: all reals

Range: $y \geq -4.3$

Relative minimum: $(-1.9, -4.3)$, $(1.1, 1.95)$

Relative maximum: $(0, 4)$

Interval(s) where function values are increasing: $-1.9 < x < 0$, $x > 1.1$

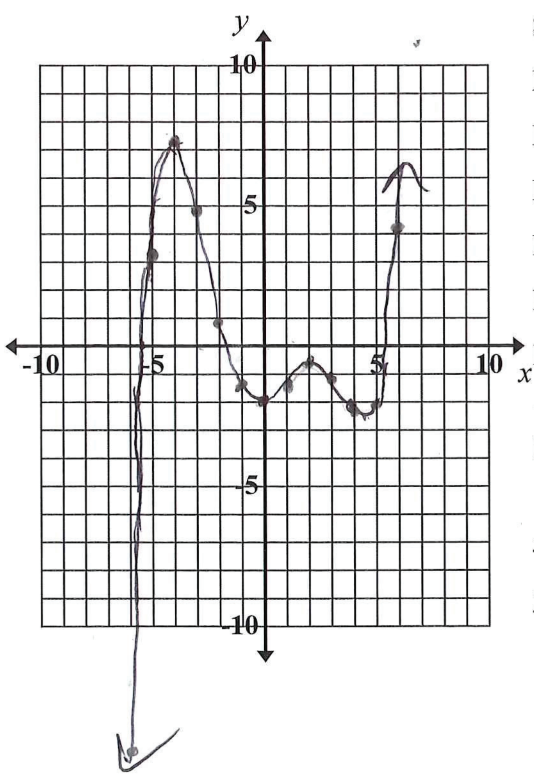
Interval(s) where function values are decreasing: $x < -1.9$, $0 < x < 1.1$

x-intercepts(s): $(-2.4, 0)$, $(-1, 0)$

y-intercept(s): $(0, 4)$

11. $y = \frac{1}{150}x^5 - \frac{1}{50}x^4 - \frac{1}{5}x^3 + \frac{3}{5}x^2 + \frac{3}{10}x - 2$

x	y
-6	-16.8
-5	3.2
-4	7.3
-3	4.7
-2	0.9
-1	-1.5
0	-2
1	-1.3
2	-0.7
3	-1.1
4	-2.3
5	-2.2
6	4.1



Sign of the Lead Coefficient: positive

End behavior: ↙ ↗

Domain: all reals

Range: all reals

Relative minimum: $(-0.23, -2.03)$, $(4.5, -2.6)$

Relative maximum: $(-4.1, 7.3)$, $(2.2, -0.7)$

Interval(s) where function values are increasing: $x < -4.1$, $-.23 < x < 2.2$, $x > 4.5$

Interval(s) where function values are decreasing: $-4.1 < x < -.23$, $2.2 < x < 4.5$

x-intercepts(s): $(-5.3, 0)$, $(-1.7, 0)$, $(5.5, 0)$

y-intercept(s): $(0, -2)$

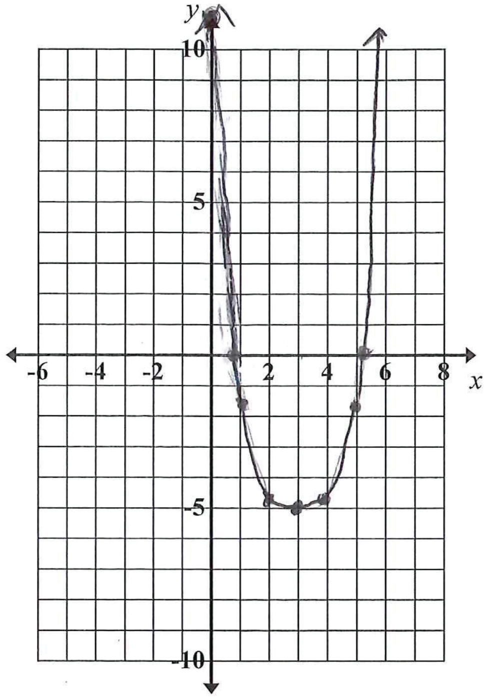
Name ______________________ Period ________

6.1B Graphing Polynomial Functions: Significant Features

#12 – 13: Use a table of values to graph each equation and identify the significant features of the graph.

12. $y = \frac{1}{5}(x-3)^4 - 5$

x	y
-1	46.2
0	11.2
1	-1.8
2	-4.8
3	-5
4	-4.8
5	-1.8
6	11.2
7	46.2



Sign of the Lead Coefficient: positive

End behavior: ↖ ↗

Domain: all reals

Range: $y \geq -5$

Relative minimum: $(3, -5)$

Relative maximum: none

Interval(s) where function values are increasing: $x > 3$

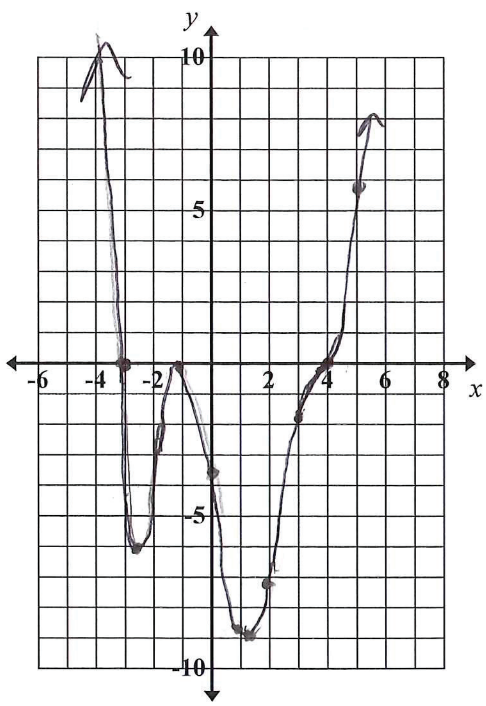
Interval(s) where function values are decreasing: $x < 3$

x-intercepts(s): $(.8, 0)$, $(5.2, 0)$

y-intercept(s): $(0, 11.2)$

13. $y = \frac{1}{50}(x+3)(x+1)^2(x-4)^3$

x	y
-4	92.2
-3	0
-2	-4.32
-1	0
0	-3.8
1	-8.6
2	-7.2
3	-1.9
4	0
5	5.8



Sign of the Lead Coefficient: positive

End behavior: ↖ ↗

Domain: all reals

Range: $y \geq -8.96$

Relative minimum: $(-2.45, -6.21)$, $(1.29, -8.96)$

Relative maximum: $(-1, 0)$

Interval(s) where function values are increasing: $-2.45 < x < -1$, $x > 1.29$

Interval(s) where function values are decreasing: $x < -2.45$, $-1 < x < 1.29$

x-intercepts(s): $(-3, 0)$, $(-1, 0)$, $(4, 0)$

y-intercept(s): $(0, -3.84)$

14. How does the degree of the polynomial function affect the end behavior of its graph?

Even degree: end beh is either ↖ ↗ or ↙ ↘

Odd degree: end beh is either ↙ ↗ or ↖ ↘

Name ______________________________ Period __________

6.1B Graphing Polynomial Functions: Significant Features

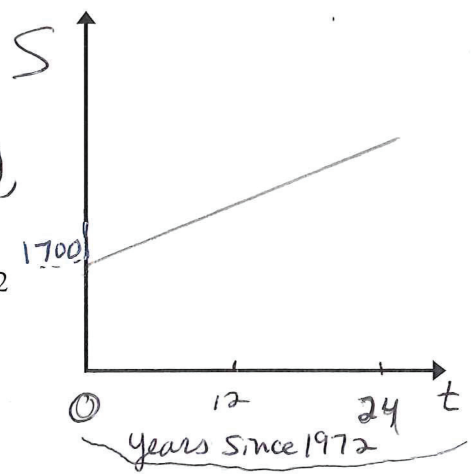
15. The retail space in shopping centers in the United States from 1972 to 1996 can be modeled by

$$S = -0.0068t^3 - 0.27t^2 + 150t + 1700$$

where S is the amount of retail space (in millions of square feet) and t is the number of years since 1972.

a) How much retail space was there in 1990? Record your thinking.

4272.9 ft^2 (millions)
$t = 18$, since 1990 is 18 years since 1972



b) Is the amount of retail space increasing or decreasing in 1995? Record your thinking. Increasing;
When $22 < t < 24$ (before and after 1995), S increases.

c) Is the amount of retail space decreasing at any point between 1972 and 1996? Record your thinking. NO
When $0 < t < 24$, S increases each year.

d) What are the domain and range of the function? What do they represent in the context of the problem?

Domain $0 \le t \le 24$, t is # years since 1972 up to 1996.
Range $1700 \le S \le 5050.5$; S is the retail space (in millions ft^2) over that time period.

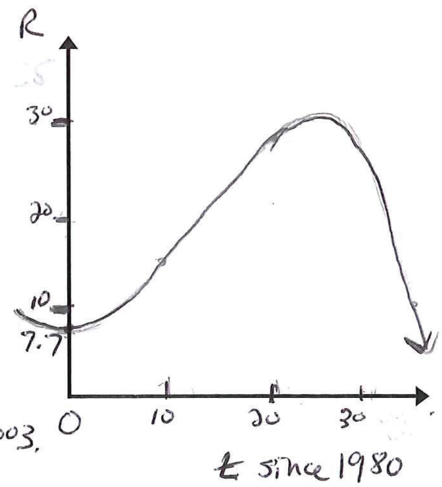
16. The average monthly cable TV rate from 1980 to 2003 can be modeled by

$$R = -0.0036t^3 + 0.13t^2 - 0.073t + 7.7$$

where R is the monthly rate (in dollars) and t is the number of years since 1980.

a) What are the domain and range of the function? What do they represent in the context of the problem?

Domain $0 \le t \le 23$; # years from 1980 to 2003
Range $\$7.70 \le R \le \$30.99$; monthly rate for cable TV in that time period



b) Why isn't this model useful past 2003? (What would your cable bill be now if you used this model?) In the year 2003, the max. rate was \$31.06 (23.8, 31.06). After that, the rates decrease over time ... not the case! For 2014, this model would yield a rate of \$14/mo.!

c) When is the graph increasing? What does this mean in the context of the problem? $0 \le t \le 23$
Cable TV Rates were always increasing from 1980 to 2003.

d) When is the graph decreasing? What does this mean in the context of the problem?
When $t > 23.8$ That the rates started declining in the year 2003.

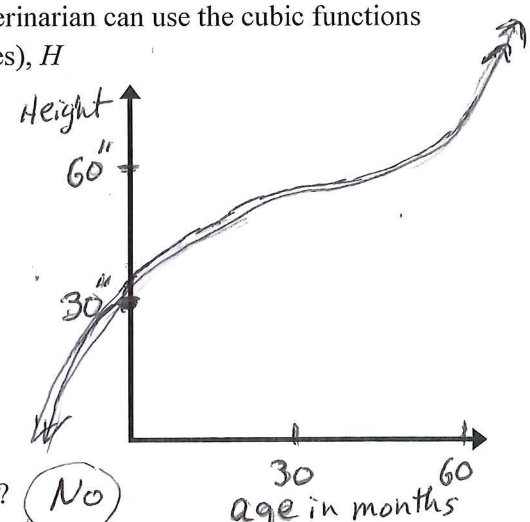
e) What is the average monthly cost of a cable bill in 1983?

$R(3) = \$8.55$

Name ____________________ Period ________

6.1B Graphing Polynomial Functions: Significant Features

17. To determine whether a Holstein heifer's height is normal, a veterinarian can use the cubic functions included below, where L is the minimum normal height (in inches), H is the maximum normal height (in inches), and t is the age (in months).



$$L = 0.0007t^3 - 0.061t^2 + 2.02t + 30$$

$$H = 0.001t^3 - 0.08t^2 + 2.3t + 31$$

a) What is the normal height range for an 18 month old heifer? Record your thinking. $L(18) = 50.7$ in

$H(18) = 52.3$ in

$50.7'' < \text{normal height} < 52.3''$

b) Can this model be used for the entire life of the heifer? Why? No

After a certain age, a heifer does not get any taller.

c) Suppose a veterinarian examines a Holstein heifer that is 43 inches tall. About how old do you think the heifer is? Explain how you got your answer. $6.6 \text{ months} < \text{age} < 8.3 \text{ months}$

Draw a horizontal line $y = 43$ and use calc. intersect on Both curves.

$L(8.3) = 43$

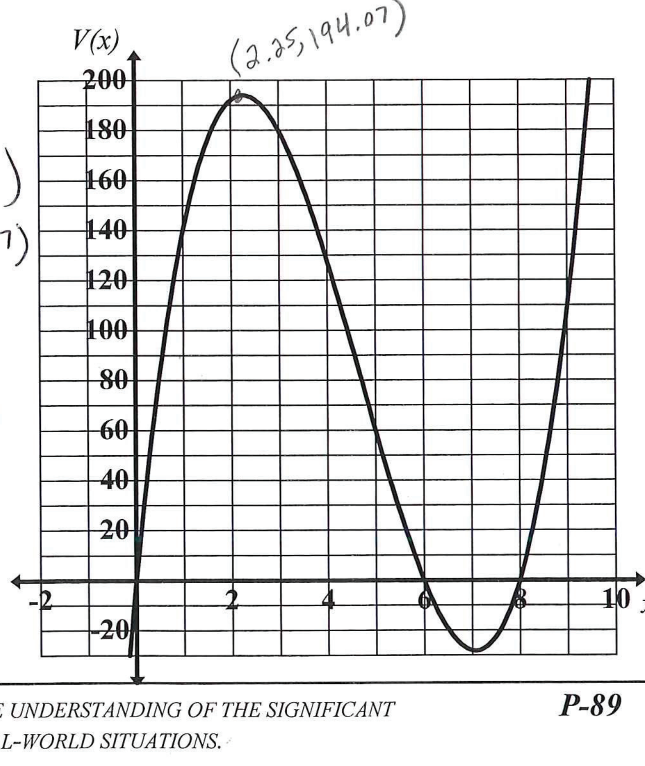
$H(6.6) = 43$

d) At what age does this model no longer work? (Hint: graph both functions at once)

At the point of inflection, the increasing height starts to level off before the graph starts to increase again. This occurs when the height is near 55", So probable age would be 30 months < age months < 31.6 months

18. A container company is making an open box from a 12 inch by 16 inch piece of metal, by cutting equal squares from each corner. The volume of the box can be modeled by the following equation and graph.

$$V(x) = (12 - 2x)(16 - 2x)(x)$$



a) Explain the restrictions on the domain and range of this model.

Domain $0 < x < 6$ (width > 0, $12 - 2x > 0 \Rightarrow x < 6$)

Range $0 < V < 194.07$ (volume > 0 and within given domain, the max Volume is 194.07)

b) What is the approximate maximum volume for this box? Explain your thinking. 194 in^3

Within the acceptable domain $0 < x < 6$, the max Volume occurs at (2.26, 194.07)

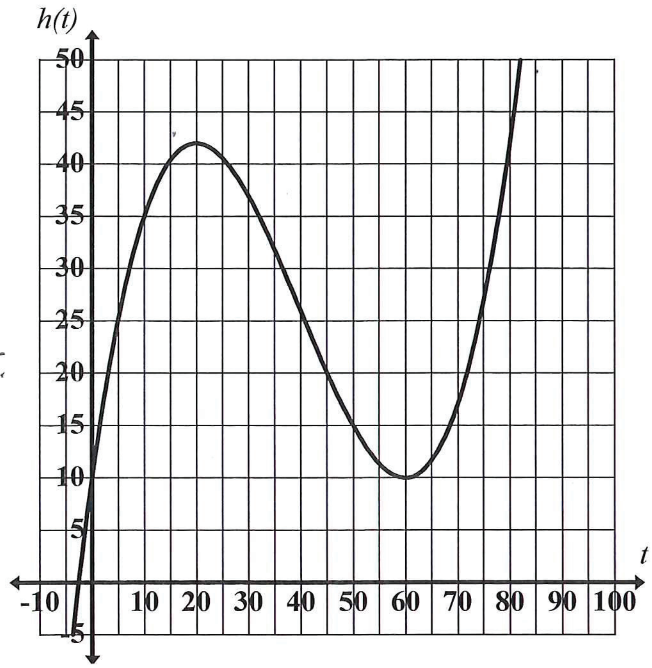
c) What size squares should they cut off of each corner in order to maximize the volume of the box? (approximate) $\approx 2\frac{1}{4}''$ squares

Name ______________________________ Period __________

6.1B *Graphing Polynomial Functions: Significant Features*

19. The function $h(t) = 0.001t^3 - 0.12t^2 + 3.6t + 10$ gives the height, *h*, (in feet) during the time *t* (in seconds) of a portion of the track of a roller coaster.

a) What is the *y*-intercept? What does this point represent in the context of the problem?



y-int = 10;
The roller coaster is 10 ft above ground before it starts a certain portion of the track.

b) Is there a relative maximum? If yes, explain what it means in the context of the problem.

(20, 42) ⟹ after 20 seconds, the roller coaster reaches its max ht. of 42 feet.

c) Is there a relative minimum? If yes, explain what it means in the context of the problem.

(60, 10) ⟹ After 60 seconds, the roller coaster returns to a height of 10 feet.

d) Does the track ever touch the ground? How can you tell?

No; the height never equals 0 after the coaster begins rolling.

e) Use the graph to find *h*(5). Explain what it means in the context of the problem.

h(5) = 25.1 After 5 seconds, the roller coaster has reached a height of 25.1 ft above the ground.

f) At what *t*-value is *h*(*t*) = 35? Explain what this means in the context of the problem.

At t = 10 secs and at t = 32.1 secs
The roller coaster's height of 35' is obtained twice during the ride — once on the way up and once again on its way down.
At t = 77, the height is again 35 feet, but the roller coaster ride lasts 60 seconds. 77 seconds is not part of the domain.

g) The ride lasts 60 seconds. Why would this model not be useful after 60 seconds?

After 60 seconds, the height of the coaster is continually increasing to ∞.

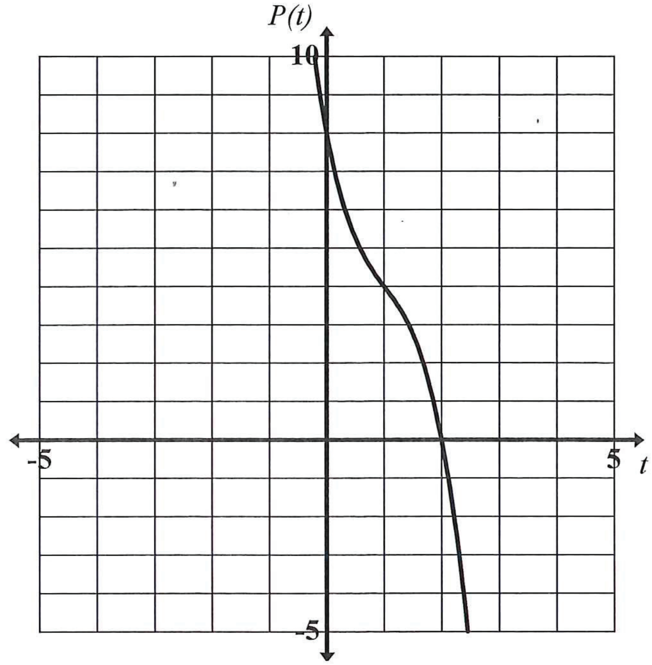
Name ____________________ Period __________

6.1B *Graphing Polynomial Functions: Significant Features*

20. A patient is receiving a certain medication in the hospital. The amount of drug (milligrams) in his bloodstream t days after the drug is taken can be modeled by the function $P(t) = -2t^3 + 6t^2 - 8t + 8$.

a) Use the graph to find how much of the drug was in the patient's bloodstream when his blood was tested 24 hours after he was given the drug.

4 mg



b) Tell the doctor how many hours it will take for the drug to be completely eliminated from his bloodstream.

2 days

c) Explain any restrictions on the domain and range for this model.

Domain $0 \le t \le 2$

After 2 days, the drug is completely out of the patient's bloodstream ($y=0$)

Range $0 \le P(t) \le 8$

8 mg is the original amount of the drug in the patient's bloodstream which lessens each hour afterwards.

Section 6.1B

Name ______________________ Period ________

6.1B *Graphing Polynomial Functions: Significant Features*

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